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Codimension zero superembeddings

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Abstract

Superembeddings which have bosonic codimension zero are studied in 3,4 and 6 dimensions. The worldvolume multiplets of these branes are off-shell vector multiplets in these dimensions, and their self-interactions include a Born-Infeld term. It is shown how they can be written in terms of standard vector multiplets in flat superspace by working in the static gauge. The action formula is used to determine both Green-Schwarz type actions and superfield actions.

1 Introduction

The superembedding formalism provides a powerful and systematic method for deriving the dynamics of super p-branes. A detailed review of the subject is given in [1] where a comprehensive list of references can be found. There is a natural constraint on such an embedding, namely that the odd tangent space of the brane should be a subspace of the odd tangent space of the target superspace at all points on the brane. It was noted in [2] that this constraint is applicable to all branes, including those which have worldvolume multiplets containing gauge fields such as D-branes and the M5-brane, and that it determines a worldvolume multiplet which can be one of three distinct types.

The three types of multiplet are: (a) on-shell multiplets, that is the basic embedding condition determines the dynamics, (b) off-shell multiplets where the embedding condition leads to a recognisable off-shell multiplet for which a superfield action can be written down, and (c) underconstrained multiplets. By underconstrained we mean that, although the worldvolume multiplet is off-shell, the resulting multiplet cannot be used to construct a Lagrangian. In the case where the target space has 32 supersymmetries (and the worldvolume 16) the multiplets are mostly of type (a) but occasionally of type (c), the latter situation occurring for low (bosonic) codimension. For fewer supersymmetries the worldvolume multiplets are more often of type (b), but again type (c) multiplets arise for low codimension. For type (c), an additional constraint is required, and in all cases studied so far it turns out that it is sufficient to impose the so-called $\mathcal{F}$-constraint, that is, one introduces an extra gauge field on the worldvolume and then constrains all of its components to vanish except for the purely bosonic ones. (For type (a) or (b) such gauge fields, if present, do not have to be introduced independently). If one considers systems of branes the $\mathcal{F}$-constraint can be derived from considerations of branes ending on other branes [3, 4, 5]. In [6] a detailed study was made of superembeddings with codimension one, where the worldvolume multiplet is an unconstrained scalar superfield. Except for the case of the membrane in $D = 4$, which is type (b), these superembeddings are of type (c) and so it is necessary to impose the $\mathcal{F}$-constraint. Alternatively, one can impose further constraints directly on the superembedding which are equivalent to the $\mathcal{F}$-constraint, but it is simpler to impose the latter directly as it is unambiguous. An example of this is given by the 5-brane in $D = 7$ for which the equations were obtained by an additional geometrical constraint in [7] and then by the $\mathcal{F}$-constraint in [6].

One can also derive actions starting from the superembedding formalism. Green-Schwarz type actions for branes can be obtained using the generalised action principle [8] which was reinterpreted in [9] as a constructive principle. For multiplets of type (a) one can also find superfield actions, the first model studied, the superparticle in $D = 3$ being an example of this [10] (see also [11] and [12] for other early papers). A third possibility, also requiring type (a) multiplets, is to construct actions in the static gauge. This has been done, for example, for the supermembrane in $N = 1, D = 4$ superspace indirectly in [13], using partially broken supersymmetry systematically in [14] and more recently, starting from the superembedding approach in [15].

In this paper we consider superembeddings with bosonic codimension zero for spacetime dimensions three, four and six. The worldvolume multiplets are off-shell Maxwell multiplets in

these dimensions and consist of a spinor field, a Maxwell field strength tensor and zero, one or three auxiliary scalars respectively combined together in a spinor superfield which can be identified with the transverse fermionic coordinate. The case of the D9-brane in ten dimensions has been studied previously in [16]; it differs from the others in the series in that the worldvolume multiplet is on-shell. A special feature of codimension zero is that, for these embeddings, the standard embedding condition imposes no constraint at all; one can always make a choice of the odd tangent bundle of the brane in such a way that it sits inside the odd tangent bundle of the target restricted to the brane. However, it still turns out that the $\mathcal{F}$-constraint is sufficient to generate the required off-shell multiplets.

It is quite simple to understand how this arises. Since the standard embedding condition imposes no constraints on the worldvolume multiplet, the latter must be an unconstrained spinor superfield corresponding to the transverse fermionic coordinate. One then introduces a new worldvolume Maxwell field with modified field strength $\mathcal{F}$. In each of the three cases the standard constraint in flat superspace is that the odd-odd part of the field strength should vanish, so if we impose this (on $\mathcal{F}$) there will be an off-shell Maxwell multiplet in addition to the spinor superfield. If we now require that the even-odd component of the field strength should vanish as well we eliminate one of the spinor fields, and thereby equate the (fermionic) Goldstone field of the superembedding with the field strength superfield of the Maxwell multiplet. Moreover, it is the unique covariant constraint which has this property.

The notion of a brane in a flat superspace is directly related to the notion of partial breaking of supersymmetry (specifically by one-half) [17]. In a superspace context this idea can be implemented using the group-theoretical method of non-linear realisations for supersymmetry [18]. It is related to the superembedding formalism in that the former can be derived from the latter by working in a suitable gauge, and in some recent papers [15] membranes and the $N = 2, D = 2$ superparticle have been discussed from this point of view. An advantage of the superembedding formalism is that it can be applied to arbitrary target superspaces, although, as in the case of the κ-symmetric Green-Schwarz formalism [25], the presence of branes may lead to constraints on the target superspace.

The non-linear realisation method has been applied to many examples; see, for example [13, 19, 20, 21, 22, 23, 24]. In the present context its relevance is that the worldvolume theory should then presumably be supersymmetric Born-Infeld theory in superspace. This has been studied in a number of papers, for example, [26, 27, 28, 29]. In [30], some partial results were given for higher-dimensional Yang-Mills theories, and a general analysis of the ten-dimensional superspace Bianchi identities which should be compatible with Born-Infeld theory has also appeared [31]. Some of these results have included work on non-Abelian extensions of Born-Infeld and this is one of the main motivations for the current study. The hope is that by discussing the theory in a superembedding context one might be able to gain some insight into how to derive the non-Abelian generalisation which would be the right one for branes (although it is not known whether it would be unique). A discussion of this problem from the point of view of κ-symmetry has been given in [32]. In fact, in the current paper, we shall not have much to say about the non-Abelian case. However, in order to be able to study this problem from the superembedding point of view, it is necessary to understand the abelian case first. Even here our results, obtained by going

to the static gauge, are as yet incomplete. We are able to derive superspace Lagrangians for $N = 1, D = 3$ and $D = 4$, but we have not yet made direct comparisons with the known results mentioned above. In order to do this, it is necessary to implemement the field redefinitions relating the different formalisms; work on this is in progress. The $D = 6$ case is yet more complicated, and we also hope to study this problem in more detail in the near future.

One aspect that we are able to comment on is the geometry induced on the worldvolume of the brane. For the $D = 3$ case this is not terribly exciting, but for $D = 4$ we find that there is an induced chiral supergeometry for which the Ogievetsky-Sokatchev potential $H^{\alpha\dot{\alpha}}$ [33] (see also [34]), or rather its deviation from the flat case, is reminiscent of the supercurrent multiplet of the worldvolume gauge supermultiplet. One might anticipate that there could be a harmonic superpace extension of this result in the $D = 6$ case where the corresponding supergravity potential structure is also known [35].

2 Superembeddings

We consider a superembedding $f : M \to \underline{M}$. Our index conventions are as follows; coordinate indices are taken from the middle of the alphabet with capitals for all, Latin for bosonic and Greek for fermionic, $M = (m, \mu)$, tangent space indices are taken in a similar fashion from the beginning of the aplphabet so that $A = (a, \alpha)$. The distinguished tangent space bases are related to coordinate bases by means of the supervielbein, $E_M{}^A$, and its inverse $E_A{}^M$. Coordinates are denoted $z^M = (x^m, \theta^\mu)$. We use exactly the same notation for the target space but with all of the indices underlined. Indices for the normal bundle are denoted by primes, so that $A' = (a', \alpha')$.

The embedding matrix is the derivative of f referred to the preferred tangent frames, thus

$$E_A{}^{\underline{A}} := E_A{}^M \partial_M z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} \tag{1}$$

The basic embedding condition is

$$E_\alpha{}^{\underline{a}} = 0 \tag{2}$$

To see the content of this constraint we can consider a linearised embedding in a flat target space in the static gauge. This gauge is specified by identifying the coordinates of the brane with a subset of the coordinates of the worldvolume, so that

$$x^{\underline{a}} = \begin{cases} x^a \\ x^{a'}(x, \theta) \end{cases} \tag{3}$$

$$\theta^{\underline{\alpha}} = \begin{cases} \theta^\alpha \\ \theta^{\alpha'}(x, \theta) \end{cases} \tag{4}$$

Since

$$E^{\underline{a}} = dx^{\underline{a}} - \frac{i}{2} d\theta^{\underline{\alpha}} (\Gamma^{\underline{a}})_{\underline{\alpha}\,\underline{\beta}} \theta^{\underline{\beta}} \tag{5}$$

in flat space, it is easy to see that, to first order in the transverse fields, the embedding condition implies

$$D_\alpha X^{a'} = i(\Gamma^{a'})_{\alpha\beta'} \theta^{\beta'} \tag{6}$$

where

$$X^{a'} = x^{a'} + \frac{i}{2} \theta^\alpha (\Gamma^{a'})_{\alpha\beta'} \theta^{\beta'} \tag{7}$$

The worldvolume multiplet is therefore described by a set of scalar superfields equal in number to the bosonic codimension and obeying the constraint (6). Clearly, for codimension one, one has a single otherwise unconstrained scalar superfield, which gives rise to a type (c) multiplet in general. For codimension zero, there are no transverse scalars, and so one has an unconstrained spinor superfield $\theta^{\alpha'}(x, \theta)$.

In the non-linear theory it is useful to study this multiplet in a covariant fashion using the geometrical quantities that are available. In order to do this it is first of all necessary to parametrise the odd-odd part of the superembedding matrix $E_\alpha{}^{\underline{\alpha}}$. Preferred bases for the odd tangent bundle $\underline{F}$ will be acted on by a group $\underline{G}$, which is either the spin group or a product of the spin group with an internal symmetry group. Without loss of generality we can write

$$E_\alpha{}^{\underline{\alpha}} = u_\alpha{}^{\underline{\alpha}} + h_\alpha{}^{\beta'} u_{\beta'}{}^{\underline{\alpha}} \tag{8}$$

where u is an element of the group $\underline{G}$ which will depend on the brane coordinates in general. In other words, we split the odd target space basis into two, with the same dimension, but allow this splitting to depend on where we are on the brane. We can also parametrise the even-even part of the embedding matrix in terms of the Lorentz transformation corresponding to the spin transformation in u; so we can choose

$$E_a{}^{\underline{a}} = u_a{}^{\underline{a}} \tag{9}$$

At this stage there is still a local $\underline{G}$ symmetry; that is, if we transform $u \mapsto gu, g \in \underline{G}$, the frame E_α will go into itself up to linear transformation as long as we also transform $h \mapsto h'$ where

$$h'_\alpha{}^{\beta'} = (-g_\alpha{}^{\gamma'} + g_\alpha{}^\delta h_\delta{}^{\gamma'})(g_{\gamma'}{}^{\beta'} - g_{\gamma'}{}^\epsilon h_\epsilon{}^{\beta'})^{-1} \tag{10}$$

In other words, h transforms projectively under $\underline{G}$. This symmetry was discussed for the 3-brane in six dimensions in [6]; it can be used to choose different gauges for $E_\alpha{}^{\underline{\alpha}}$.

The field h was first introduced in [2] and plays a crucial röle whenever there are gauge fields on the brane. It is related in a non-linear fashion to the field strength of the gauge field for D-branes, for example. For scalar branes h vanishes except if there are auxiliary scalar fields.

3 Static gauge

It is not difficult to extend the linearised analysis presented above to the non-linear case when the target space is flat. This has been discussed in some examples in [], and shows how the superembedding formalism is related to the non-linear realisation formalism [].

We can always choose coordinates, at least locally on the brane, in which the odd frame has the form

$$E_\alpha = A_\alpha{}^\beta(D_\beta + \psi_\beta{}^b\partial_b) \tag{11}$$

where, since we are discussing flat space, we no longer need to distinguish coordinate indices. We may take the even basis vectors to be

$$E_a = B_a{}^b\partial_b \tag{12}$$

The matrices A and B are included for convenience in order to facilitate comparison with the covariant approach.

The dual form bases are

$$\begin{aligned} E^\alpha &= e^\beta(A^{-1})_\beta{}^\alpha && (13)\\ E^a &= (e^b - e^\beta\psi_\beta{}^b)(B^{-1})_b{}^a && (14) \end{aligned}$$

where e^a, e^α denote the standard bases of flat superspace,

$$\begin{aligned} e^\alpha &= d\theta^\alpha && (15)\\ e^a &= dx^a - \frac{i}{2}d\theta^\alpha(\Gamma^a)_{\alpha\beta}\theta^\beta && (16) \end{aligned}$$

If we then pull back the target space preferred frames and express the result in terms of the frame E^A, the basic embedding condition gives rise to a number of results. Firstly, we find that the field $\psi_\alpha{}^a$ is given by

$$\psi_\alpha{}^a = \frac{i}{2}D_\alpha\theta'\Gamma^b\theta'(\delta_b{}^a - \frac{i}{2}\partial_b\theta'\Gamma^a\theta')^{-1} \tag{17}$$

This expresion essentially determines the induced geometry on the brane. We also find the non-linear version of (6); it is

$$\mathcal{D}_\alpha X^{a'} = i(\Gamma^{a'})_{\alpha\beta'}\theta^{\beta'} \tag{18}$$

where

$$\mathcal{D}_\alpha := D_\alpha + \psi_\alpha{}^a \partial_a \tag{19}$$

with $X^{a'}$ given by (7) as before. For the embedding matrix we find

$$E_\alpha{}^{\underline{\beta}} \to \begin{cases} E_\alpha{}^\beta = A_\alpha{}^\beta \\ E_\alpha{}^{\beta'} = A_\alpha{}^\gamma \mathcal{D}_\gamma \theta^{\beta'} \end{cases} \tag{20}$$

as well as

$$E_a{}^{\underline{b}} \to \begin{cases} E_a{}^b = B_a{}^c(\delta_c{}^b - \frac{i}{2}\partial_c\theta'\Gamma^b\theta') \\ E_a{}^{b'} = B_a{}^c\partial_c X^{b'} \end{cases} \tag{21}$$

and

$$E_a{}^{\underline{\beta}} \to \begin{cases} E_a{}^\beta = 0 \\ E_a{}^{\beta'} = B_a{}^b\partial_b\theta'^{\beta'} \end{cases} \tag{22}$$

The matrices A and B can then be determined by comparing with the covariant forms for the embedding matrix so that

$$u_\alpha{}^\beta + h_\alpha{}^{\gamma'} u_{\gamma'}{}^\beta = A_\alpha{}^\beta \tag{23}$$

$$u_\alpha{}^{\beta'} + h_\alpha{}^{\gamma'} u_{\gamma'}{}^{\beta'} = A_\alpha{}^\gamma \mathcal{D}_\gamma \theta^{\beta'} \tag{24}$$

while

$$u_a{}^b = B_a{}^c(\delta_c{}^b - \frac{i}{2}\partial_c\theta'\Gamma^b\theta') \tag{25}$$

$$u_a{}^{b'} = B_a{}^c\partial_c X^{b'} \tag{26}$$

For the codimension zero case we have the same set of equations with the difference that those equations involving a' indices are no longer present. Since the bosonic tangent spaces for brane and target are the same, it is permissible to take $u_a{}^b = \delta_a{}^b$, and so we find an explicit expression for B in terms of the field $\theta'(x,\theta)$:

$$B_a{}^b = (\delta_a{}^b - \frac{i}{2}\partial_a\theta'\Gamma^b\theta')^{-1} \tag{27}$$

The corresponding spin group matrix is also trivial but u with odd indices need not be because of the presence of an internal symmetry group.

4 Codimension zero

It is well-known that the super Maxwell multiplets in $D = 3, 4, 6$ and 10 dimensions are associated with the division algebras $\mathbb{K} = \mathbb{R}, \mathbb{C}, \mathbb{H}$ and $\mathbb{O}$ respectively [36, 37]. The codimension zero embeddings in these dimensions can be understood in this light, too, particularly for the cases we discuss in this paper, namely $D = 3, 4, 6$. In all four cases we have superembeddings of an $N = 1$ worldvolume superspace into an $N = 2$ target superspace. The spin group in these dimensions can be viewed as $SL(2, \mathbb{K})$, the internal symmetry group of the target space is $U(2, \mathbb{K})$ and the internal symmetry group of the worldvolume is $U(1, \mathbb{K})$, although the internal symmetry groups for $D = 10$ do not seem to fit into this pattern. Translated into more standard language, the internal symmetry groups are $SO(2), U(2)$ and $USp(4)$ for $N = 2$ in $D = 3, 4$ and 6 respectively, and $1, U(1)$ and $USp(2) = SU(2)$ for $N = 1$. In the following we shall use standard notation for each of the three cases, so that spinors are real with two-components in $D = 3$, complex with two-components in $D = 4$, and pseudo-Majorana-Weyl with eight components subject to a reality condition in $D = 6$.

In view of the fact that we may choose the even-even part of the embedding matrix to be the unit matrix for codimension zero, and take the Lorentz group factor of the group element u which occurs in the embedding matrix to be 1, the odd tangent space basis vectors on the brane can be written

$$E_\alpha = v_1{}^i E_{\alpha i} + h_\alpha{}^\beta v_2{}^j E_{\beta j} \tag{28}$$

where $i = 1, 2$ for $D = 3, 4$, and

$$E_{\alpha i} = v_i{}^I E_{\alpha I} + h_{\alpha i}{}^{\beta j'} v_{j'}{}^J E_{\beta J} \tag{29}$$

where $i = 1, 2$, and $I = 1, 2, 3, 4$ for $D = 6$. In both of these formulae, the basis vectors on the right-hand side are standard frames for the target space which we shall take to be flat for simplicity throughout this paper. The matrix v, composed of $(v_1{}^i, v_2{}^i)$ for $D = 3, 4$ and of $(v_i{}^I, v_{i'}{}^I)$ for $D = 6$ is an element of the target space internal symmetry group. Strictly, in equation (28), the spinor indices should be four-component with the spinors obeying a pseudo-Majorana-Weyl condition in the $D = 4$ case, but when we have imposed the $\mathcal{F}$-constraint we shall see that it can be interpreted in terms of two-component complex spinors.

The task now is to compute the dimension zero torsion on the brane,

$$T_{\alpha\beta}{}^c = E_\alpha{}^{\underline{\alpha}} E_\beta{}^{\underline{\beta}} T_{\underline{\alpha}\,\underline{\beta}}{}^c \tag{30}$$

and then to introduce $\mathcal{F}$ satisfying the modified Bianchi identity

$$d\mathcal{F} = -\underline{H} \tag{31}$$

where $\underline{H}$ denotes the pull-back of a closed three-form on the target superspace, $\underline{H} = d\underline{B}$. One then imposes the $\mathcal{F}$-constraint

$$\mathcal{F}_{\alpha\beta} = \mathcal{F}_{\alpha b} = 0 \tag{32}$$

and analyses (31) at dimension zero. This completes the determination of the multiplet as a non-linear super-Maxwell multiplet, and shows how the fields h and v in (28) or (29) are related to the dimension zero fields in the Maxwell multiplets, i.e. the field strength tensors and auxiliary scalars. To find the action it then only remains to specify the Wess-Zumino $D+1$-form.

4.1 D=3

For dimension three we choose the gauge $v = 1$ in (28) so that

$$E_\alpha = E_{\alpha 1} + h_\alpha{}^\beta E_{\beta 2} \tag{33}$$

with

$$h_{\alpha\beta} = \epsilon_{\alpha\beta} k + (\gamma^a)_{\alpha\beta} h_a \tag{34}$$

The target space dimension zero torsion is

$$T_{\alpha i \beta j}{}^c = -i\delta_{ij}(\gamma^c)_{\alpha\beta} \tag{35}$$

using which we find the worldvolume dimension zero torsion is

$$T_{\alpha\beta}{}^a = -i(\gamma^b)_{\alpha\beta} m_b{}^a \tag{36}$$

where

$$m_{ab} = f\eta_{ab} - 2h_a h_b - 2\epsilon_{abc} k h^c \tag{37}$$

with

$$f = 1 + k^2 + h^2 \tag{38}$$

The closed target space three-form H can be chosen to be such that its only non-vanishing component (in flat space) is

$$H_{\alpha i \beta j c} = -i(\gamma_c)_{\alpha\beta}(\tau_1)_{ij} \tag{39}$$

where τ_1 is the first Pauli matrix. When the $\mathcal{F}$-constraint has been imposed the dimension zero component of the $\mathcal{F}$ Bianchi identity (31) reads

$$T_{\alpha\beta}{}^{b}\mathcal{F}_{bc} = E_{\alpha}{}^{\underline{\alpha}} E_{\beta}{}^{\underline{\beta}} H_{\underline{\alpha}\,\underline{\beta} c} \tag{40}$$

where $\mathcal{F}_{ab} = \epsilon_{abc}\mathcal{F}^{c}$. From this we find that $k = 0$ so that $f = 1 + h^2$, and also that

$$\mathcal{F}_a = \frac{2h_a}{(1+h^2)} \tag{41}$$

As promised, therefore, the $h_\alpha{}^\beta$ field in the embedding matrix is non-linearly related to the field strength tensor of the Maxwell multiplet.

4.2 D=4

For $D = 4$ we can choose $\underline{H}$ to be such that its only non-vanishing component is

$$H_{\alpha i \dot{\beta} c}{}^{j} = -i(\tau_1)_i{}^j(\sigma_c)_{\alpha\dot{\beta}} \tag{42}$$

where τ_1 again denotes the first Pauli matrix. If we use the gauge in which $v = 1$, it is straightforward to show that the embedding is chiral in the sense that, in two-component notation, there is no $h_\alpha{}^{\dot{\beta}}$. We simply have

$$E_\alpha = E_{\alpha 1} + h_\alpha{}^\beta E_{\beta 2} \tag{43}$$

and similarly for the dotted odd basis vectors which are obtained by complex conjugation. Using this in the $\mathcal{F}$ identity one can show that $h_\alpha{}^\alpha$ is subject to one real algebraic constraint while $h_{(\alpha\beta)}$ is proportional to $\mathcal{F}_{\alpha\beta}$, so that the embedding describes the correct degrees of freedom of the off-shell $N = 1, D = 4$ Maxwell supermultiplet. (Here $\mathcal{F}_{\alpha\beta}$ refers to the self-dual part of $\mathcal{F}_{ab}$ in spinor notation.) However, for computational purposes it is easier to switch to the gauge where $h_\alpha{}^\alpha = 0$, which requires the introduction of a non-trivial v. If we do this and then impose the $\mathcal{F}$-constraint we find, after making a choice of $U(1)$ gauge ($U(1)$ being the internal symmetry group of the worldvolume), that

$$v = \exp(\frac{iu}{2}\tau_1) \tag{44}$$

as well as

$$h_{\alpha\beta} = A\mathcal{F}_{\alpha\beta} \tag{45}$$

where A is a complex function which satisfies

$$1 + f^2 A\bar{A} = -\bar{A} \tag{46}$$

where $\mathcal{F}_\alpha{}^\gamma\mathcal{F}_{\gamma\beta} := \epsilon_{\alpha\beta}f^2$. In this gauge the auxiliary field is u while A is determined in terms of $\mathcal{F}_{ab}$ from (46).

4.3 D=6

For $D = 6$ it is again convenient to choose a gauge in which v is non-trivial. Implementing the $\mathcal{F}$-constraint, where

$$H_{\alpha I \beta J} = -i(\gamma)_{\alpha\beta} H_{IJ} \tag{47}$$

and with

$$H_{IJ} = \begin{pmatrix} 0 & \epsilon_{ij'} \\ \epsilon_{i'j} & 0 \end{pmatrix} \tag{48}$$

we find

$$E_{\alpha i} = v_i{}^J E_{\alpha J} + h_\alpha{}^\beta \delta_i{}^{j'} v_{j'}{}^J E_{\beta J} \tag{49}$$

where

$$v = \exp\, i u_r \begin{pmatrix} 0 & \tau_r \\ \tau_r & 0 \end{pmatrix} \tag{50}$$

where $\tau_r, r = 1, 2, 3$ are the Pauli matrices. The three fields u_r can be identified as the triplet of auxiliary fields while $h_\alpha{}^\beta$ is a non-linear function of $\mathcal{F}_{ab}$.

5 Actions

In this section we discuss Green-Schwarz actions and superfield actions for codimension zero branes. We concentrate on the $D = 3$ case for simplicity adding some comments on the other cases at the end.

We recall that the Wess-Zumino form W_{D+1} is a closed $D+1$-form which can be written in the explicit form $W_{D+1} = dZ_D$ where Z_D is a potential D-form constructed from given target space fields and $\mathcal{F}$. On the brane it is exact and so can be written $W_{D+1} = dK_D$. The Lagrangian form is $L_D = K_D - Z_D$ and is closed by construction. The Green-Schwarz action for the brane is

$$S_{GS} = \int d^D x\, \epsilon^{m_1 \ldots m_D} L_{m_1 \ldots m_D}(x, \theta = 0) \tag{51}$$

where the integration is taking over the bosonic worldvolume M_o. A simple argument shows that this action is κ-symmetric and reparametrisation invariant on M_o [9]. This procedure works for all branes, provided that the worldvolume is not type (c), with the exception of those whose worldvolume multiplets contain self-dual tensor fields for which other techniques are available [38]. In the case where the worldvolume multiplet is off-shell one can construct superfield actions using two different approaches. The first method involves imposing the embedding condition

by a Lagrange multiplier, as in the superparticle in $D=10$ [39] or the heterotic string [40], for example. Presumably this approach can be generalised to codimension zero, but here we shall focus on the second approach which can be used to derive an interacting superfield Lagrangian for the worldvolume multiplet in the static gauge. Superfield actions in the static gauge have been derived for membranes in [13, 14, 15].

5.1 D=3

For $D=3$ the Wess-Zumino form for our choice of H_3 is

$$W_4 = \underline{G}_4 + \underline{G}_2 \mathcal{F} \tag{52}$$

where $\underline{G}_2$ is a closed target space two-form and $\underline{G}_4$ satisfies

$$d\underline{G}_4 = \underline{G}_2 \underline{H}_3 \tag{53}$$

we therefore have

$$\begin{aligned} \underline{G}_2 &= d\underline{C}_1 && (54) \\ \underline{G}_4 &= d\underline{C}_3 - \underline{C}_1 \underline{H}_3 && (55) \end{aligned}$$

where $\underline{C}_1$ and $\underline{C}_3$ are potential forms and $\underline{H}_3$ being the three-form appearing in the $\mathcal{F}$ Bianchi identity. In flat target space the non-vanishing components of the $\underline{G}$-forms are

$$\begin{aligned} G_{\alpha i \beta j} &= -i\epsilon_{\alpha\beta}\epsilon_{ij} && (56) \\ G_{\alpha i \beta j cd} &= -i(\gamma_{cd})_{\alpha\beta}(\tau_3)_{ij} && (57) \end{aligned}$$

As noted above W_4 can be written as dK_3. It is straightforward to check that the only non-vanishing component of K_3 is the purely even one $K_{abc} = \epsilon_{abc}K$, where

$$K = \frac{(1-h^2)}{(1+h^2)} \tag{58}$$

Moreover, given the relation (41) between h_a and $\mathcal{F}_a$, it is easy to check that K is the Born-Infeld Lagrangian

$$K = \sqrt{(\eta_{ab} + \mathcal{F}_{ab})} \tag{59}$$

To compute the kinetic part of the action one then has to convert to a coordinate basis using $E_m{}^a$ evaluated at $\theta = 0$. This gives a factor of the determinant of this bosonic worldvolume dreibein, which is the dreibein for the usual GS metric. The GS action is then completed by the

Wess-Zumino term. However, for codimension zero it is possible to arrange for $E_m{}^\alpha$ to vanish, so that one only needs to find Z_{abc} in order to find the Wess-Zumino term. The even-even part of the worldvolume supervielbein is

$$E_m{}^a = \delta_m{}^b (B^{-1})_b{}^a \tag{60}$$

so that in any gauge the Green-Schwarz Lagrangian for the system is

$$L_{GS} = \big(\det (B^{-1}) L\big) \, | \tag{61}$$

where $L_{abc} := \epsilon_{abc} L$, and where the bar denotes evaluation of a superfield at $\theta = 0$.

The superfield Lagrangian is more difficult to compute. We use the method advocated in [41] to relate x space actions to closed D-forms in superspace and work in the static gauge discussed in section 3 with the matrix A set equal to the unit matrix. The basis forms are then

$$\begin{aligned} E^a &= (e^b - e^\beta \psi_\beta{}^b)(B^{-1})_b{}^a &(62)\\ E^\alpha &= e^\alpha &(63) \end{aligned}$$

where $e^A = (e^a, e^\alpha)$ are the standard basis forms for $N = 1, D = 3$ flat superspace. The idea is now to identify a component of the three-form L_3 as the superspace Lagrangian and then to show that the resulting x-space action is the same as the GS action. This is most easily done by working in the flat basis e^A, so we shall write the components of L_3 in this basis as ℓ_{ABC}, whereas its components in the E^A basis will be denoted by L_{ABC}.

Now, if the Lagrangian three-form is changed by the addition of an expression of the form dX_2, where X_2 is some two-form, the action will be unaltered, so we can make use of this freedom to change L_3 such

$$\ell_{\alpha\beta\gamma} = 0 \tag{64}$$

(Note that this is not the case for the original L_3.) This is always possible by a suitable choice of X_{ab} since $(dX)_{\alpha\beta\gamma}$ includes a term of the form $(\gamma^d)_{(\alpha\beta} X_{\gamma)d}$; in fact, $\ell_{\alpha\beta\gamma}$ can be made equal to zero by such a transformation where X_{ab} is gamma-traceless. Since the new L is still closed it is straightforward to check that this implies, for an appropriate choice of X_{ab},

$$\ell_{\alpha\beta c} = (\gamma_c)_{\alpha\beta} L_o \tag{65}$$

It is L_o which is to be identified with the superspace Lagrangian. To prove that this is correct we first note that the scalar part is unaffected by the change of L_3 by dX_2 due to the gamma-tracelessness of X_{ab} (It is also assumed that $X_{\alpha\beta} = 0$.) Secondly, it is easy to show that the top component of L, i.e. $\ell_{abc} := \epsilon_{abc}\ell$, is the top component of the scalar superfield L_o and is thus the x-space Lagrangian for the superfield action

$$S_{SF} = \int d^3x\, d^2\theta\, L_o, \tag{66}$$

i.e. $\ell = D_\alpha D^\alpha L_o$. Now, under the change $L_3 \to L_3 + dX_2$, ℓ_{abc} does change, but by $\partial_{[a} X_{bc]}$ and this is irrelevant in the (x-space) action. On the other hand it is easy to see that

$$L_{abc} = B_a{}^d B_b{}^e B_c{}^f \ell_{def} \tag{67}$$

from which we find that $\ell = \det(B^{-1}) L = L_{GS}$ when evaluated at $\theta = 0$ as claimed.

To compute it explicitly it is necessary to make gauge choices for the $\underline{C}$ fields. Normally one would do this treating $\theta^{\alpha 1}$ and $\theta^{\alpha 2}$ on an equal footing. However, in the present context we do not wish to have explicit worldvolume $x's$ or $\theta's$ appearing in the Lagrangian, so it is better to choose a gauge in which the potentials only depend on θ^2, which becomes the brane superfield, and which we shall denote by $\Lambda(x, \theta)$.

For the three-form potential $\underline{C}_3$ we can choose a gauge in which the non-vanishing components are

$$\begin{aligned}
C_{\alpha 1 \beta 1 c} &= (\gamma_c)_{\alpha\beta} (\theta^2)^2 && (68)\\
C_{\alpha 2 \beta 2 c} &= -(\gamma_c)_{\alpha\beta} (\theta^2)^2 && (69)\\
C_{\alpha 2 bc} &= i(\gamma_{bc})_{\alpha\beta} \theta^{\beta 2} && (70)\\
C_{abc} &= \epsilon_{abc} && (71)
\end{aligned}$$

where $\theta^{\alpha 2}\theta^{\beta 2} := \epsilon^{\alpha\beta}(\theta^2)^2$. For $\underline{C}_1$ we have

$$C_{\alpha 1} = i\theta^2_\alpha \tag{72}$$

Given these choices it is straightforward to find the Lagrangian three-form and compute its components with respect to the flat basis. To find the superfield Lagrangian we need

$$L_o \propto (\gamma^c)^{\alpha\beta} \ell_{\alpha\beta c} \tag{73}$$

A shortish computation yields

$$L_o = \Lambda^2 \left(1 - \frac{1}{2} D_\alpha \Lambda_\beta D^\alpha \Lambda^\beta\right)^{-1} \tag{74}$$

This final Lagrangian is in agreement with that derived in [14] using partially broken supersymmetry (after a field redefinition).

5.2 D=4,6

For $D = 4, 6$ deriving the Green-Schwarz actions is a similar procedure. In $D = 4$ the Wess-Zumino form is

$$W_5 = \underline{G}_5 + \underline{G}_3\mathcal{F} \tag{75}$$

where $d\underline{G}_5 = \underline{G}_3\underline{H}_3$. The non-vanishing components of the $\underline{G}$'s are

$$G_{\alpha i\dot{\beta}c}{}^{j} = -i(\sigma_c)_{\alpha\dot{\beta}}(\tau_2)_i{}^j \tag{76}$$
$$G_{\alpha i\dot{\beta}cde}{}^{j} = -2(\sigma_{cde})_{\alpha\dot{\beta}}(\tau_3)_i{}^j \tag{77}$$

Using the results of subsection 4.2 it is relatively straightforward to show that $K_{abcd} := \epsilon_{abcd}K$ is the only non-vanishing component of K_4 and that it has the explicit form

$$K = \cos u\sqrt{-\det\left(\delta_a{}^b + \mathcal{F}_a{}^b\right)} \tag{78}$$

so that, on eliminating u, we recover the standard Born-Infeld form.

For $D = 6$ we have

$$W_7 = \underline{G}_7 + \underline{G}_5\mathcal{F} + \frac{1}{2}\underline{G}_3\mathcal{F}^2 \tag{79}$$

where

$$d\underline{G}_7 = \underline{G}_5\underline{H}_3 \tag{80}$$
$$d\underline{G}_5 = \underline{G}_3\underline{H}_3 \tag{81}$$

The non-vanishing components of the target space forms are

$$G_{\alpha I\beta Jc} = (\gamma_c)_{\alpha\beta}G^3_{IJ} \tag{82}$$
$$G_{\alpha I\beta Jcde} = (\gamma_{cde})_{\alpha\beta}G^5_{IJ} \tag{83}$$
$$G_{\alpha I\beta Jcdefg} = \epsilon_{cdefgh}(\gamma^h)_{\alpha\beta}G^7_{IJ} \tag{84}$$

where the matrices G^3 , G^5 , G^7 are

$$G^3_{IJ} = i\begin{pmatrix} \epsilon_{ij} & 0 \\ 0 & -\epsilon_{i'j'} \end{pmatrix} \tag{85}$$

$$G^5_{IJ} = i\begin{pmatrix} 0 & \epsilon_{ij'} \\ -\epsilon_{i'j} & 0 \end{pmatrix} \tag{86}$$

$$G^7_{IJ} = i \begin{pmatrix} \epsilon_{ij} & 0 \\ 0 & -\epsilon_{i'j'} \end{pmatrix} \tag{87}$$

The $USp(4)$ invariant metric is

$$\eta_{IJ} = \begin{pmatrix} \epsilon_{ij} & 0 \\ 0 & \epsilon_{i'j'} \end{pmatrix} \tag{88}$$

In this case one can again verify $W_7 = dK_6$, and that the only non-vanishing compoent of K_6 is the purely even one. We anticipate that we should again find the Born-Infeld Lagrangian dressed by the auxiliary scalar fields.

As in the $D = 3$ case we can construct superfield actions although they are no longer full superspace integrals. It is easy to see, for $D = 4, 6$, that L_D does not contain a scalar component of the right dimension to be a candidate Lagrangian for such an action.

The $D = 4$ case was studied in detail for a general theory in [41]. In flat superspace, for example, given a closed four-form L_4 satisfying the constraints

$$\begin{aligned} L_{\alpha\beta\gamma\delta} &= 0 & (89) \\ L_{\alpha\beta\gamma d} &= 0 & (90) \end{aligned}$$

one can find a chiral Lagrangian in the component $L_{\dot\alpha\dot\beta cd}$ which includes a term of the form $(\sigma_{cd})_{\dot\alpha\dot\beta}L_o$. The real part of the top component of this form is the purely even part of L_4, i.e. L, where $L_{abcd} = \epsilon_{abcd}L$, so that $L = D^2 L_o + \bar{D}^2\bar{L}_o$. Again one is allowed to change L_4 by dX_3 for some three-form X_3. To apply this to the brane case, we can use a similar argument to the the $D = 3$ case to show that the chiral action constructed from $L_4 = K_4 - Z_4$ in the flat basis e^A reproduces the Green-Schwarz action.

In the $D = 6$ case, the superfield action we are able to construct relatively easily is an example of a superaction [42], discussed for $D = 6$ supersymmetric Yang-Mills theory in [43]. This time, in flat space, if all the lower components in L_6 vanish one can show that

$$L_{abcd\alpha i\beta j} = (\gamma_{[abc})_{\alpha\beta}L_{d]ij} \tag{91}$$

The field L_{aij} (symmetric on ij) is the candidate Lagrangian; it is to be integrated with respect to the "measure" $d^6x\, D^i_\alpha D^j_\beta(\gamma^a)^{\alpha\beta}$. The resulting action is supersymmetric due to the fact that $dL_6 = 0$. For the brane we anticipate that the argument given above for the $D = 3$ and $D = 4$ cases will work here as well, although the full details remain to be worked out.

6 Induced geometry

In this section we discuss some aspects of the supergeometry induced on the brane for codimension zero embeddings. In the case of $D = 3$, the choice of bases we made earlier led to a

non-standard form for the worldvolume dimension zero torsion. However, we can easily bring it to standard form by a change of even basis using the matrix $m_a{}^b$. One can then modify E_a and fix the $SL(2,\mathbb{R})$ connection on the brane to obtain the standard form of $D=3$ supergeometry. Essentially the only constraints one imposes in $D=3$ are conventional ones. In the embedding framework, the geometry is therefore completely standard and the induced supergravity potential is the field $\psi_\alpha{}^b$ which is given explicitly in terms of the transverse fermion field Λ_α.

For $D=4$ the situation is more interesting because, as we have seen, the $\mathcal{F}$-constraint enforces chirality. In two-component notation the even target space basis forms are

$$E^a = dx^a + \frac{i}{2} d\theta^{\alpha i}(\sigma^a)_{\alpha\dot{\alpha}}\bar{\theta}_i^{\dot{\alpha}} + \frac{i}{2} d\bar{\theta}_i^{\dot{\alpha}}(\sigma^a)_{\alpha\dot{\alpha}}\theta^{\alpha i} \tag{92}$$

Pulling this back to the worldvolume in the gauge where $v=1$ and exploiting chirality we find

$$\mathcal{D}_\alpha \bar{\Lambda}^{\dot{\beta}} = 0 \tag{93}$$

where

$$\mathcal{D}_\alpha := D_\alpha + \psi_\alpha{}^a \partial_a \tag{94}$$

as before, with

$$\psi_\alpha{}^a = -\frac{i}{2}(D_\alpha \Lambda \sigma^b \bar{\Lambda} + D_\alpha \bar{\Lambda}\sigma^b \Lambda)(\delta_b{}^a + \frac{i}{2}\partial_b \Lambda \sigma^s \bar{\Lambda} + \frac{i}{2}\partial_b \bar{\Lambda}\sigma^a \Lambda)^{-1} \tag{95}$$

From these equations we see that the chirality constraint on Λ given in (93) is highly non-linear. However, after a little algebra (and using the fact that Λ is chiral) we find that $\psi_\alpha{}^a$ can be rewritten as

$$\psi_\alpha{}^a = i D_\alpha J^b (\delta_b{}^a - i\partial_b J^a)^{-1} \tag{96}$$

where

$$J^a := -\frac{1}{2}\Lambda \sigma^a \bar{\Lambda} \tag{97}$$

From this, we find that the odd basis vector on the worldvolume is

$$E_\alpha = D_\alpha + i D_\alpha J^b (\delta_b{}^a - i\partial_b J^a)^{-1}\partial_a \tag{98}$$

This is reminiscent of the form of E_α in Ogievetsky-Sokatchev supergravity obtained by transforming from special chiral coordinates to standard coordinates. To see this we recall that we can complexify an $N=1, D=4$ superspace M with a chiral structure to M_L, say, and then use Frobenius' theorem to write the continuation of the dotted basis as

$$E_{\dot\alpha} = -\frac{\partial}{\partial\varphi_L^{\dot\alpha}} \tag{99}$$

in adapted coordinates $(x_L, \theta_L, \varphi_L)$. These coordinates are related to the analytic continuation of the coordinates of M by

$$\begin{aligned} x_L^a &= x^a + iH^a(x,\theta,\bar\theta) && (100)\\ \theta_L^\alpha &= \theta^\alpha && (101)\\ \varphi_L^{\dot\alpha} &= \bar\theta^{\dot\alpha} && (102) \end{aligned}$$

where H^a is the Ogievetsky-Sokatchev potential, and $\bar\theta$ becomes the complex conjugate of θ when we return to the real superspace. When we express $E_{\dot\alpha}$ in terms of $(x,\theta,\bar\theta)$ and return to real superspace we find (taking the complex conjugate)

$$E_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\partial_\alpha H^b(\delta_b{}^a - i\partial_b H^a)^{-1} \tag{103}$$

This is the same as (98) provided that we set

$$H^{\alpha\dot\alpha} = -\frac{1}{2}\theta^\alpha\bar\theta^{\dot\alpha} + J^{\alpha\dot\alpha} \tag{104}$$

Note that the first term in this expression is just the potential for flat superspace. In other words the induced Born-Infeld geometry for $D=4$ is an Ogievetsksy-Sokatchev geometry with potential equal to the flat potential plus a term which looks to be the supercurrent for the Maxwell multiplet.

7 Conclusions

In this paper we have described some aspects of superembeddings with bosonic codimension zero in $D=3,4,6$. In particular, we have discussed how the worldvolume super-Maxwell multiplets arise when the $\mathcal{F}$-constraint is imposed and we have seen how one can construct Green-Schwarz type actions and superfield actions in the static gauge. Further calculations will be necessary in order to have a completely satisfactory description of these theories as superembeddings, particularly from the point of view of comparing the results that can be derived from this formalism with superspace Born-Infeld theory.

One task that we have not carried out in this paper is to find the worldvolume superspace field strength in the static gauge. If we write $\mathcal{F} = F - \underline{B}_2$, then F satisfies a normal Bianchi identity $dF=0$. Its components can be computed quite straightforwardly knowing $\underline{B}_2$ and $\mathcal{F}$. Again, it is preferable to choose a gauge for $\underline{B}_2$ on the target space such that its components depend only on θ^2 and not on x or θ^1. This is always possible, although the resulting F is complicated. Moreover, the field strength superfield Λ is not the standard one that one would use in flat

superspace. For $D = 4$, for example, the field Λ is covariantly chiral, whereas the usual field strength is ordinarily chiral, as it is in [26]. However, it is not difficult to construct a chiral field $\lambda = \Lambda + \ldots..$ and this will be discussed in more detail elsewhere. (The relation between the chiral and covariantly chiral spinor superfields has been discussed in the context of partially broken supersymmetry in [19].)

One generalisation that can be made concerns the target space geometry. In this paper we have made the simplest choices possible for the various forms that are needed, i.e. the $\underline{G}'s$ and $\underline{H}'s$, but it is possible to treat these in a more systematic fashion. The sets of forms that arise in this way presumably reflect the supergravity theories which can provide consistent backgrounds for these objects.

Finally, we have noted that the induced geometry for $D = 4$ turns out to have a simple interpretation in terms of the Ogievetsky-Sokatchev potential. As we remarked earlier, it would be interesting to see if this could be generalised to the $D = 6$ case where one might suspect that harmonic superspace methods could come into play.

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Note added

In a recent preprint some similar results have been derived for the D=4 case [44].

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